SYNOPSIS

Title : "SOME PROBLEMS ON NON-STANDARD STRUCTURE OF AXIOMATIC SET THEORY AND ITS IMPACT"

(a) Introduction

We propose to develop an axiomatic framework for non-standard set theory by reformulating the Zermelo–Fraenkel axioms through the enlargement method based on internal sets, an approach initially explored by G. Kreisel for philosophical purposes. E. Nelson later introduced Internal Set Theory to formalize A. Robinson's infinitesimal analysis. Building on Nelson's formulation, we aim to create an alternative axiomatization of Zermelo–Fraenkel set theory with the Zermelo–Fraenkel Axiom of Choice using the tools of infinitesimal calculus. We extend the concept of consistency of continuum hypothesis derive by K. Godel. Further, we intend to extend Lindström's transfer principle to study the implications of the saturation and standardization axioms, incorporating quantifier-reduction rules and Fehrele's principle. We also propose to investigate the consequences of consistency proofs for Zermelo-Fraenkel set theory. Gödel's Continuum Hypothesis, which remains an open problem, continues to provoke debate in axiomatic set theory. Our aim is to explore the consistency of Zermelo–Fraenkel through a method analogous to Gentzen's cut-elimination procedure in first-order logic.

(b) Work Plan

Kreisel and Wong obtained important results concerning paradoxes in set theory from both formalist and intuitionistic perspectives. In addition, we plan to analyze the algebraic structures underlying set theory through the lens of category theory as developed by Saunders Mac Lane, focusing on its meta-mathematical foundations. Tarski's model-theoretic treatment of sets-parallel to Fraenkel's axiomatic system but without the Axiom of Infinity—and the von Neumann–Bernays school's consideration of inaccessible ordinals, which distinguish the Axiom of Choice from the Axiom of Regularity, will form part of our comparative study. Finally, we seek to analyze A. Robinson's theory of non-standard sets in relation to Leibniz's original framework of infinitesimals.

(c) Details of the project with chapterisation

Mathematical logic serves as a foundational mirror for mathematics. Abraham Robinson developed non-standard analysis, a framework capable of addressing the completeness theorem, and we aim to establish connections between this theory and the elementary structure of Boolean-valued models in set theory. The concept of Boolean-valued models, originating from the work of Church and Sikorski, has been effectively applied across various branches of mathematics, including both non-standard and non-commutative settings. Mac Lane's foundational work in category theory provides an effective framework for developing algebraic structures and for examining Gödel's completeness theorem within set theory. We aim to compare category theory with axiomatic set theory to highlight their structural and foundational relationships. Additionally, we intend to analyze the structure of Bernays–Gödel set theory, regarded as a conservative extension of Zermelo–Fraenkel set theory. Our broader objective is to develop a unified foundation that connects set theory with classical predicative logic.

The generalization of classical predicate calculus and its model-theoretic interpretations has historically given rise to various paradoxes-beginning with Cantor, and later studied by Russell, Richard, and Burali-Forti-prompting intensive work toward establishing consistency and completeness within Zermelo–Fraenkel set theory due to its extensive applications in pure and applied mathematics. Gödel and Cohen developed models involving bounded quantifiers to address consistency questions, and we seek to further develop such model-theoretic structures using enlargement techniques and infinitesimal calculus, particularly through filters, ultrafilters, and forcing. The proposed research project is classified into four chapters, reflecting its broad applications across various allied fields of mathematics. The chapters are as follows:

Chapter 1	Axiomatic set theory and its nonstandard interpretation
Chapter 2	Characterization of Nonstandard measure theory
Chapter 3	Impact of Godel's completeness theorem and its applications
Chapter 4	The Syntactic and Semantic Framework of Set Theory

(d) Hypothesis

We propose to investigate Cantorian perspectives in set theory alongside Hilbert's program, particularly with respect to strongly inaccessible numbers and Mahlo axioms. A central focus of this study is the analysis of the consistency of elementary number theory through Gentzen's cut-elimination theorem. The concept of proper classes plays a crucial role in set theory, especially in the context of transfinite induction. The principles underlying set construction, as formulated in the Zermelo–Fraenkel axioms, appear both natural and indispensable.

Shelah's work on cardinal arithmetic introduced new algebraic structures, such as monoids, ultrafilters, and ultraproducts, with significant applications to lattices and Boolean-valued models of set theory. We propose to analyze various notions of cofinality under the consistency assumptions of the Continuum Hypothesis and the Axiom of Choice within generalized Zermelo–Fraenkel frameworks. Robinson and Kreisel developed non-standard formulations to address paradoxes using monoids, filters, and ultrafilters, while Nelson introduced Internal Set Theory to refine Robinson's non-standard analysis. Our project aims to build upon these foundational insights in a systematic and integrative manner.

(e) Methodology

The proposed research project is purely theoretical in nature. Relevant books and reference materials required for its preparation are available in the Central Library of B. N. M. University. Additional resources, including recent books and journals related to the research topic, may be consulted at the National Library (Science Division) in Kolkata and the Indian Statistical Institute.

(f) References

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Signature of the Supervisor

Baban Kumar

Signature of the candidate

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UNIVERSITY DEPTT. OF MATHEMATICS H.O.D

Signature of the Head of the University Department